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Supplementary Materials for

Is low fertility really a problem? Population aging, dependency, and consumption

Ronald Lee,* Andrew Mason,* members of the NTA Network

*Corresponding author. E-mail: rlee@demog.berkeley.edu (R.L.); amason@hawaii.edu (A.M.)

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Materials and Methods

Fiscal support ratio (FSR)

FSRs are presented in Table S1 for countries for 29 countries for which requisite data are available. The FSRs are calculated for each country using the age profiles of taxes, tax(x), and public transfer inflows, tgi(x), in the base year and population by age for case z, N(x,z). Case z refers to a population age distribution that conforms to a particular set of assumptions. The FSRs are normalized by setting the FSR(2010), the FSR calculated using the estimated 2010 population, to 1 and calculating values for other cases relative to the value calculated for 2010:

$$FSR(z) = \frac{\sum_{x=0}^{\omega} taxes(x)N(x,z)}{\sum_{x=0}^{\omega} tgi(x)N(x,z)} \Bigg/ \frac{\sum_{x=0}^{\omega} taxes(x)N(x,2010)}{\sum_{x=0}^{\omega} tgi(x)N(x,2010)}. \quad (1)$$

The normalized FSR are presented in Table S1 because they are easier to interpret. The normalization procedure does not affect any results.

Support ratio (SR)

SRs are presented in Table S2 for countries for which requisite data are available. The SRs are calculated for each country using the age profiles of labor income, yl(x), and consumption, c(x), in the base year and the population age for case z, N(x,z). The labor income values are normalized by dividing by the simple average of per capita labor income at each age from 30 to 49. Consumption values are normalized in the same fashion using consumption by single year of age for those 30 to 49.

$$SR(z) = \frac{\sum_{x=0}^{\omega} \big(yl(x)/yl(30-49) \big) N(x,z)}{\sum_{x=0}^{\omega} \big(c(x)/c(30-49) \big) N(x,z)}. \qquad (2)$$

NTA Data

NTA age profiles for labor income, consumption, taxes, public transfer inflows and outflows, and private transfer inflows and outflows are provided on the NTA website (http://ntaccounts.org/web/nta/show/Science). The NTA data are constructed as nominal values in each country's own currency and the values are presented that way on the website. The values were constructed for a recent year avoiding periods of economic crisis or other forms of atypical conditions to the extent possible.

Detailed methods are presented in the UN National Transfer Accounts Manual (*11*). Four NTA profiles are used in the analysis:

Labor income: The value of the work effort of employees, the self-employed and unpaid family workers. Labor income is measured by earnings, the value of employer-provided benefits and an estimate of labor's share of income from unincorporated business. It also includes a portion of indirect taxes less subsidies.

Consumption: The value of goods and services consumed by the public and private. Taxes on products, e.g., value added tax, is not included in the value of consumption.

Taxes: Compulsory, unrequited payments, in cash or in kind, made by the private sector to the public sector including social contributions. Public transfer inflows refer to taxes used to cover the costs of public transfer inflows.

Public transfer inflows: Cash and in-kind transfers to households from all levels of government including public pensions, public education, publicly funded health programs and compulsory national health insurance, and all other public spending on goods and services.

Age profiles are estimated using household surveys and administrative records from public agencies responsible for collecting revenues and providing public services. Age-profiles for spending on pensions, health, and education are constructed using detailed information from surveys combined with utilization information, e.g., health utilization or school enrollment by education level. Simple equivalence scales are using to allocate other household consumption, e.g., food, clothing, and housing. Profiles are scaled to match aggregate economic values as part of the System of National Accounts produced by statistical agencies in each country.

NTA were constructed by research teams from each country. The teams are based in universities or, in a few cases, national statistical agencies. The research teams vary in their composition but often include researchers with expertise in economics, demography, and economic statistics. All research teams have participated in multiple training workshops held in regional centers in Africa, Asia, Europe, and Latin America or held at the East-West Center in Honolulu or the Center for the Economics and Demography of Aging at University of California at Berkeley. The results have been thoroughly reviewed by experienced NTA researchers based in Berkeley or Honolulu.

Supplementary Text

The analysis is concerned with the long-run implications of demographic variables for macroeconomic outcomes, such as the state of public finances or the material standard of living.

Population, age structure, fertility and mortality

Consider a population closed to migration in which age specific survival rates from birth to age x, l(x), and age specific fertility rates, f(x), are constant over time. Mortality and fertility conditions are summarized by life expectancy at birth, $e_0 = \int l(x)dx$ and the total fertility rate, $TFR = \int f(x)dx$. The population will reach a steady state with a fixed population age structure and a constant population growth rate, n. The size of the population *per se* has no implications for any outcomes, because we rely on standard economic models in which scale has no effect on per capita values. We set the number of births in arbitrary year t (we will suppress time subscripts) to equal one without loss of generality. The total population in year t is then given by:

$$N = \int_0^{\omega} e^{-nx} l\left(x\right) dx \qquad (3)$$

where $e^{-nx} l(x)$ is the population at each age. The exponential term captures differences in the size of the birth cohort to which individuals at age x belong, while l(x) is the proportion of each birth cohort that survives to age x. For a growing population, older members will belong to smaller birth cohorts than younger members and, hence, the population will be younger.

The mean age of the population, A, provides a useful summary measure of population age structure and is defined as:

$$A = \int_0^{\omega} x e^{-nx} l\left(x\right) dx \Big/ \int_0^{\omega} e^{-nx} l\left(x\right) dx. \qquad (4)$$

For a given survival schedule and any age distribution of childbearing, there is a one-to-one mapping between the population growth rate and the TFR in steady-state:

$n \approx \ln\left(\frac{1}{2.05} l_\mu TFR\right) \Big/ \mu$, where 1/2.05 is the female share of births, μ is the average age at which women give birth in the stable population, and l_μ is the proportion of births surviving to μ. Typically, μ is around 30 years and this value is used for calculations. When the sex ratio at birth deviates from 2.05, the approximation must be adjusted accordingly. It is clear from inspection that an increase in the fertility rate (TFR) leads to more rapid population growth and to a younger population.

Equivalence scales and effective populations

Changes in age structure have important economic effects because ability, needs, and behavior vary with age. This age variation is captured using an empirically-based equivalent adult scale for each kind of economic variable. For example, the equivalent adult scale for labor income varies by age reflecting age variation in labor force participation, unemployment rates, hours worked, and wages or labor productivity per hour. The equivalent adult scale for consumption varies with biological needs, tastes, decisions about spending on children, and many other factors (*24*). In all cases we take those who are 30-49 as our reference group and the equivalence value for those at any age is expressed relative to the average for that group. For the labor income equivalence scale, for example, a value of 0.5 for twenty-five-year-olds would indicate that they earn 50% on average of those who are 30-49 years old. The equivalence scales are based on cross-sectional estimates of the age profile of interest. In all analysis the equivalence scale is assumed to remain constant over time.

Fixed equivalence scales are combined with changing population data to capture how changes in age structure affect a population aggregate of interest. For example, weighting the population by the equivalent adult scale for labor income tells us how age structure influences the number of effective workers over time. This is not a forecast of labor, but rather a calculation of the purely demographic source of change in labor as the population age distribution changes, abstracting from all other change. In general, we represent the effective population by N_z and the equivalence scale ψ_z where z is the economic activity of interest. The steady state effective population for activity z is:

$$N_z = \int_0^\omega e^{-nx} l(x) \psi_z(x)\, dx. \qquad (5)$$

The number of effective workers in the population, for example, is given by:

$$N_{y_l} = \int_0^\omega e^{-nx} l(x) \psi_{y_l}(x)\, dx. \qquad (6)$$

The "per capita" levels of economic variables are expressed relative to the effective population. For example, labor income per effective worker denoted by $\bar{y}_l$ and consumption per effective consumer by $\bar{c}$ are calculated using aggregate labor income and consumption as $\bar{y}_l = Y_l / N_{y_l}$ and $\bar{c} = C/N_c$ Because the equivalence age profile is fixed for economic variables, values at every age vary in direct proportion to changes in the level of that variable. Similarly, the maximum value at every age is realized when the value per effective population member is maximized.

The age distribution of any effective population can also be summarized by its average age. The average age for effective workers is:

$$A_{y_l} = \int_0^\omega x e^{-nx} l(x) \psi_{y_l}(x)\, dx \Big/ \int_0^\omega e^{-nx} l(x) \psi_{y_l}(x)\, dx. \quad (7)$$

Many of the results presented below depend on how a change in the population growth rate affects the size of the effective population. A useful steady state property is that the partial effect of a change in the population growth rate on the size of an effective population relative to the size of the birth cohort is given by (*25, 26*):

$$\frac{\partial \ln N_z}{\partial n} = -A_z \quad (8)$$

The partial effect of population growth on the per capita value of the effective population is:

$$\frac{\partial \ln N_z/N}{\partial n} = A - A_z. \quad (9)$$

For an activity that is concentrated late in life, for example, an increase in the population growth rate leads to a decline in the effective population relative to total population. For an activity concentrated early in life, an increase in the population growth rate leads to an increase in effective population per capita.

Public sector and public transfers

The public sector is characterized by taxes paid and benefits received at each age including all cash and in-kind transfers (public transfer inflows). The equivalent adult scales for taxes paid and public transfer inflows received are given and expressed relative to the average tax payment or benefit of persons 30-49. The number of effective tax payers is given by N_{tax} and the number of effective beneficiaries is given by N_{tgi} as defined above in equation (5). The level of taxation is measured by taxes per effective taxpayer, tax and the level of benefits is measured by benefits per effective beneficiary, tgi. Total taxes collected is equal to $N_{tax}tax$, the effective number of taxpayers multiplied the tax per equivalent taxpayer. Total benefits paid is equal to $N_{tgi}tgi$, the effective number of beneficiaries times the benefit level, benefit per effective beneficiary.

The fiscal condition of the government, *Surplus,* is measured as the natural log of the ratio of revenues to benefits $\ln N_{tax}tax/N_{tgi}tgi$. Note that if *Surplus*=0, the public transfer budget is balanced with taxes equal to benefits. The relationship between age structure and the fiscal condition in steady state is:

$$Surplus = \ln\left(tax/tgi\right) + \ln FSR \qquad (10)$$

where FSR is the fiscal support ratio, $FSR = N_{tax}/N_{tgi}$, the effective number of taxpayers per effective beneficiary.

This simple relationship can be used to assess two possible responses to changes in age structure – changes in the fiscal status of the public sector and changes in the levels and benefits required to maintain a given fiscal status. Moving the tax-benefit ratio to the other side and differentiating with respect to the population growth rate yields:

$$\frac{\partial Surplus(n)}{\partial n} + \frac{\partial \ln\left(tgi(n)/tax(n)\right)}{\partial n} = \frac{\partial \ln FSR(n)}{\partial n}. \qquad (11)$$

The impact of the population growth rate on the support ratio depends on whether the effective number of tax payers declines or increases relative to the effective number of beneficiaries as the population growth rate changes. As shown above in equation (8) an increase in population growth leads to a decline in an effective population equal to the mean age of the effective population. Noting that $\ln FSR = \ln N_{tax} - \ln N_{tgi}$ the derivative of ln FSR with respect to n is equal to the mean age of effective beneficiaries less the mean age of effective taxpayers and, thus:

$$\frac{\partial Surplus(n)}{\partial n}+\frac{\partial \ln\left(tgi(n)/tax(n)\right)}{\partial n}=A_{tgi}-A_{tax}. \qquad (12)$$

The most favorable fiscal conditions exist when the fiscal support ratio reaches its maximum. The first order condition for the population growth rate that yields $FSR^{\max}$ is:

$$\frac{\partial \ln FSR(n)}{\partial n}=A_{tgi}-A_{tax}=0. \quad (13)$$

The second order conditions are met, although we will not show them here, because beneficiaries are concentrated at young and old ages while taxpayers are concentrated in the middle ages of the age distribution.

Many of the properties of public transfers carry over to private transfers and to total transfers, the sum of private and public transfers.

The Economy: Cross-sectional and longitudinal perspectives

All economies are governed by two sets of constraints. Flow constraints apply to economic flows during any period and, simply put, say that income must be consumed, transferred, or saved. Using capital letters to represent aggregate flows, the aggregate flow constraint is:

$$Y_l+Y_A+\mathrm{T}=C+S. \qquad (14)$$

where Y_l is aggregate labor income, Y_A is aggregate asset income, T is total net transfers which is equal to aggregate net transfers from the rest of the world (because domestic inflows cancel domestic outflows), *C* is consumption, and *S* is aggregate saving. In a closed economy, total net transfers, T, will equal zero. The flow constraint holds for the public and private sectors separately, and for the total economy which is their sum. The flow constraint also applies to every age group.

The second constraint is the lifetime budget constraint that must hold for each cohort. The newborn cohort enters the world with no assets. The present value of lifetime consumption must equal the present value of lifetime labor income plus lifetime net transfers. For older cohorts, the present value of lifetime consumption must equal the present value of lifetime labor income plus the present value of lifetime net transfers plus assets.

The longitudinal labor income profile of the cohort born in the base year is given by $e^{\lambda x}\psi_{y_l}(x)\bar{y}_l$. The variable $\bar{y}_l$ is the labor income of an equivalent worker in the base year. Given steady productivity growth, labor income of an equivalent worker will grow at rate *λ*. Labor income at each age will vary depending on the equivalence scale which captures variation in labor productivity and labor supply across age relative to an equivalent worker. In similar fashion, consumption at each age over the lifetime of the newborn cohort is given by $e^{\lambda x}\psi_c(x)\bar{c}$ and net transfers at each age over the lifetime of the newborn cohort are given by $e^{\lambda x}\psi_\tau(x)\bar{\tau}$.

Designating the discount rate as *r*, the lifetime budget constraint for the newborn cohort is:

$$\int_0^\omega e^{(\lambda-r)x}l(x)\bar{y}_l\psi_{y_l}(x)dx+\int_0^\omega e^{(\lambda-r)x}l(x)\bar{\tau}\psi_\tau(x)dx=\int_0^\omega e^{(\lambda-r)x}l(x)\bar{c}\psi_c(x)dx. \qquad (15)$$

In order to simplify notation below, we define PV_z:

$$PV_z = \int_0^{\omega} e^{(\lambda - r)x} l(x) \psi_z(x) dx \qquad (16)$$

This is the lifetime present value for the new born cohort of effective labor force, or effective numbers of consumers, etc. Using this notation, the lifetime budget constraint can be rewritten as:

$$\bar{y}_l PV_{y_l} + \bar{\tau} PV_{\tau} = \bar{c} PV_c. \qquad (17)$$

An important feature of this model is the parallel between the effective population at a point in time and the present value of the effective population for the new born cohort. These two values, given in equation (5) and equation (16), are identical if the discount rate, r, is equal to the rate of growth of total income, $\lambda + n$.

Consumption loan economy

Following Samuelson (*27*) consider an economy in which all lifecycle needs are met by transfers. (Samuelson also considers reliance on a credit market and shows that transfers can achieve the Pareto optimal life cycle consumption path while a credit market cannot. We do not consider credit further here.) There is no capital and, hence, labor determines the total amount of production and income. There are no durable goods and, hence, no saving. Under these conditions, all of labor income in a given period is consumed in that period, as expressed in the cross-sectional budget constraint:

$$\bar{c} N_c = \bar{y}_l N_{y_l} \qquad (18)$$

Total consumption, consumption per equivalent consumer times the number of equivalent consumers, must equal total labor income, labor income per equivalent worker times the number of equivalent workers. The level of labor income is exogenously determined while the level of consumption is endogenous and determined by the support ratio, the effective number of workers over the effective number of consumers. Rearranging terms and defining the support ratio as $SR = N_{y_l} / N_c$, we have

$$\begin{aligned} \bar{c} &= \bar{y}_l SR \\ \ln \bar{c} &= \ln \bar{y}_l + \ln SR \end{aligned} \qquad (19)$$

In the absence of capital, introduced below, changes in population growth have no effect on labor income. Relying on equation (8) the effect of a change in the population growth rate on the level of consumption is given by:

$$\frac{\partial}{\partial n} \ln \bar{c} = \frac{\partial}{\partial n} \ln SR = A_C - A_{y_l} \qquad (20)$$

a result first derived by Arthur and McNicoll (*4*) .

The first order condition for consumption-maximizing population growth is that the mean ages of consumption and labor income are equal, $A_C = A_{y_l}$. The second order condition is met because the variance of the consumption profile is always greater than the variance in the labor income profile due to the periods of dependency at the beginning and end of life.

In the simple transfer economy there are no credit markets or interest rates, but the transfer system yields an implicit rate of return that satisfies the lifecycle budget constraint,

$$PV_{y_l} = PV_c. \qquad (21)$$

By comparing the lifecycle budget constraint to the cross-sectional budget constraint it is clear that that interest rate that satisfies the lifecycle constraint is $n+\lambda$, called the biological rate of interest by Samuelson. Aggregate consumption and the present value of lifetime consumption for the newborn cohort are identical. The population growth rate that maximizes per capita consumption also maximizes the present value of lifetime consumption.

The value of this analysis is that it shows the circumstances under which the support ratio is an accurate indicator of the effects of age structure on standards of living. In economies without capital and complete reliance on transfers to deal with lifecycle issues, an increase in the support ratio leads to an increase in the level of consumption. The maximum consumption is achieved when the maximum support ratio is realized. And that is the age structure at which the average age of the effective consumer is the same as the average age of the effective worker.

Introducing capital

Introducing capital influences consumption in two ways. First, income and possibly consumption can be raised by increasing the capital intensity of the economy. Second, increasing the capital intensity of the economy requires that some additional portion of income be devoted to saving rather than consumption. These are both captured in the cross-sectional or social budget constraint, equation (22). The left-hand-side is total consumption equal to the level of consumption times the effective number of consumers. The right-hand-side is total income less total saving with net transfers to the rest of the world assumed to be zero. Total income is calculated as the product of income, including both labor income and asset income per effective worker ($\bar{y}=y_l+y_a$), and the number of effective workers. Total saving is calculated as the product of total income and the saving rate, s, defined as saving as a share of total income.

$$\bar{c}N_c=\bar{y}N_{y_l}-s\bar{y}N_{y_l} \qquad (22)$$

Income per effective worker is endogenous and increases with capital per effective worker. But an increase in capital per effective workers comes at a cost – higher saving and lower consumption. Rearranging the cross-sectional budget constraint and gathering like terms yields:

$$\bar{c}=(1-s)\bar{y}SR. \qquad (23)$$

Consumption per effective consumer is equal to the product of 1 minus the saving rate, income per effective producer, and the support ratio.

The relationship between capital and saving is not explicit in equation (23), but follows from the well-known steady-state condition derived by Solow (*28*) :

$$s\bar{y}=(n+\lambda+\delta)k \qquad (24)$$

where δ is the rate of depreciation.

The model is completed by specifying either s or k in equation (24), which will reflect behavior, policy, and exogenous factors that will vary across countries and over time. Thus, we consider two cases that we believe span the plausible range of responses in capital and saving.

The low capital cost specification is theoretically grounded for the case of a Social Planner who seeks to maximize over an infinite horizon the socially time discounted value of per capita utility weighted by population size. Here the long run optimal trajectory holds the capital output ratio constant and the saving rate varies with the population growth rate (*18, 19*). In the long term the marginal product of capital equals the social discount rate.

In the high capital cost specification we consider a special case of the neoclassical growth model (*28*) in which the saving rate and capital-output ratio are chosen to maximize steady state per capita consumption. This is known as the “golden rule” growth path. Again referring to the Ramsey model, if the Planner’s social welfare function is not weighted by population size then eventually the marginal product of capital equals the rate of social time preference plus the population growth rate (*18, 19*). If social time preference is zero, this collapses to the golden rule case. This golden rule growth path has a higher capital-output ratio than the low capital cost case described above and is our high capital cost case.

Low capital cost case.

Holding the capital-output ratio fixed implies that the saving rate must vary in response to changes in the population growth rate – rising when the population growth rate increases and declining when the population growth rate declines. An advantage of slower population growth and population aging is that a lower saving rate will maintain the capital-output ratio and output per worker. Dividing both sides of equation (24) by income per effective consumer, we have:

$$s = (n + \lambda + \delta)\, k/\bar{y}$$

$$\frac{\partial s}{\partial n} = \frac{k}{\bar{y}}. \qquad (25)$$

An increase in the population growth rate by one percentage point must be matched by an increase in the saving rate by three percentage points to maintain the 1980-2004 average capital-output ratio of 3.0 for 14 OECD countries (*20*).

The impact of population growth on steady-state consumption per equivalent consumer is found by taking the derivative of the natural log of the cross-sectional budget constraint in equation (23) with respect to the population growth rate:

$$\frac{\partial}{\partial n}\ln \bar{c} = \frac{\partial}{\partial n}\ln(1-s) + \frac{\partial}{\partial n}\ln SR. \qquad (26)$$

The right-hand-side does not include any change in capital per effective worker, k, because capital per effective worker is constant following from the assumption that the capital-output ratio is constant. (This holds for the Cobb-Douglas production function, for example.) Hence, income per effective worker is unaffected by the change in population growth, only the portion of income that must be saved in addition to the change in the support ratio.

The first term on the right-hand-side is equal to $-\partial s/\partial n\big/(1-s)$. Substituting from equation (25) and from equation (20) we have:

$$\begin{aligned}\frac{\partial}{\partial n}\ln \bar{c} &= -\frac{k}{(1-s)\bar{y}} + A_c - A_{y_l} \qquad (27)\\ &= -K/C + A_c - A_{y_l}.\end{aligned}$$

An increase in the population growth rate has a smaller effect – less positive or more negative – on consumption per equivalent adult than is captured by the support ratio. It also follows that the consumption maximizing population growth rate is less than the support ratio maximizing population growth rate. The maximum is realized when:

$$A_c - A_{y_l} = K/C. \qquad (28)$$

Note that K/C and the mean ages of consumption and labor income vary with the population growth rate. K/C can be calculated directly from variables that are exogenous in this specification as:

$$\frac{K}{C} = \frac{K}{Y}\frac{1}{1-s}$$
$$= \frac{K/Y}{1-(n+\delta+\lambda)K/Y} \qquad (29)$$

<u>High capital cost case</u>

The golden rule case, used as the high capital cost case, has been discussed relatively extensively in the literature (*4-8*). In the golden rule case, the saving rate, capital-labor ratio, and capital-output ratio adjust to insure the maximum possible consumption per equivalent consumer. Arthur and McNicoll (4) shows that across golden rule paths

$$\frac{d\ln\overline{c}}{dn} = -\frac{K}{C} + A_c - A_{yl}. \qquad (30)$$

This is the same first order condition that holds in the fixed capital-output ratio case, but in this case the capital-output ratio adjusts to changes in the population growth rate and is generally much higher than observed capital-output ratios around the world. For the special case of a constant returns to scale Cobb-Douglas aggregate production function, with capital coefficient α, depreciation rate δ, and rate of total factor productivity growth λ, the golden rule ratio (K/C)*, is:

$$(K/C)^* = [\alpha/(1-\alpha)]/(n+\delta+\lambda) \qquad (31)$$

From Willis (5), Lee (6, *7*), and Lee and Mason (*8*, Chapter 2), we know that C times the RHS of (30) equals aggregate transfer wealth per capita (the difference between total life cycle wealth and the portion of it that is held as capital). We can conclude that the first order condition for the optimal level of fertility or population growth rate is that aggregate transfer wealth be zero, which is equivalent to the condition that the aggregate demand for life cycle wealth is exactly met by holdings of capital.

This value of n is also related to what Samuelson (*15, 16*) called the goldenest golden rule. He proved:

SERENDIPITY THEOREM. At the optimum growth rate g*, private lifetime saving will just support the most-golden golden-rule lifetime state.

This theorem states that at the optimal population growth rate g* (or n* in our notation) private saving will just support the golden rule steady state, but this is the same as saying that aggregate transfer wealth equal to zero is necessary to achieve the golden rule steady state, given the optimal life cycle planning and saving of the representative individuals in each generation. His theorem follows from our observation that derivative of life time consumption with respect to the population growth rate just equals transfer wealth, which must therefore be zero at the optimum.

This golden rule case leads to valuable insights about the economic consequences of population aging. If fertility declines to low levels the support ratio will decline, as emphasized above, but capital deepening will also occur leading to higher wages and more capital income. We see that the fertility rate that maximizes the support ratio is higher than the fertility rate that maximizes lifetime consumption when there is capital.

The golden rule case is an attractive assumption because it leads to elegant and simple results, but is obviously a very strong assumption to make. In most or all actual economies the K/C ratio is well below the golden rule level for various reasons including public sector transfers to the elderly and rates of time preference that weight present consumption highly relative to future consumption, dampening saving rates. Typical values of the K/C ratio are around 4, while for a stationary population with n=0 the golden rule ratio would be around 7 (with δ=.05 and λ=.02, with the capital coefficient α=.33).

There is no closed form solution for the consumption maximizing population growth rate. The first order conditions for all of consumption-maximizing population growth rates include endogenous variables, the mean ages of consumption and labor income, and numerical methods are used to solve for the consumption-maximizing population growth rate. The simulation model is available as part of the supplementary materials.

Immigration

Immigration and immigration policy provide another mechanism for influencing population age structure, population growth, and hence macroeconomic outcomes. In principle, immigration policy could be used to raise the fiscal support ratio by increasing the number of effective taxpayers relative to the effective number of beneficiaries or to raise the support ratio by increasing the number of effective workers relative to the number of effective consumers.

Immigration influences age structure through two channels. One is direct and depends on the age structure of the immigrant population as compared with the native population. Typically, immigrants are relatively young when they arrive so that the short-term effect of immigration, when it first begins, is to reduce the average age of the combined populations. The long-term result may be an older population, because immigrant populations age in the same way as native populations but unlike natives, they arrive at ages well past birth. Immigration indirectly influences age structure through its effects on fertility and mortality rates. These effects are not likely to be large or persistent to the extent that fertility and mortality rates are influenced by the conditions of the country of residence rather than the country of origin, and converge to host country levels relatively quickly. Taking both the direct and indirect effects of immigration into consideration, previous studies have concluded that immigration has a relatively modest impact on age structure (*12-14, 29, 30*).

A number of studies have considered whether immigration could improve public finances through an improved age structure, i.e., a higher fiscal support ratio. A targeted immigration policy that allowed immigrants to remain only so long as they were at high tax-paying ages would have favorable fiscal effects. Storesletten (*25*), for example, concludes that US fiscal conditions would be particularly favored by admitting 1.6 million 40-44-year-old high-skilled immigrants annually (net) as compared with roughly 1 million net immigrants at all ages and skill-levels combined at present. Storesletten and other studies of more realistic immigration flows conclude that fiscal effects are modest. Auerbach and Oreopoulos (*26*) conclude for the US that “even an enormous change in the rate of in-migration simulated as an outright immigration ban after the year 2000 has a small impact on fiscal balance relative to the size of the overall imbalance itself. Thus, more realistic changes in the level of immigration should be viewed neither as a major source of the existing imbalance nor as a potential solution to it”. Bonin et al (*27*) find that net migration has a positive impact on fiscal conditions in Germany particularly if they favor highly-skilled immigrants, but even then conclude “that even high immigrant inflows only partially remove the intergenerational fiscal imbalance induced by aging in Germany.”

The effect of immigration on standards of living will depend both on the age structure effects and capital cost effects associated with additional population growth. The likely magnitudes can be judged from net migration rates estimated by the UN Population Division (*1*). For the NTA countries included in our study, Jamaica had the most significant net outflow losing about one percent of its population annually between 1950 and 2010. Otherwise Mexico is the next highest with a net outflow of 0.3 percent per year. The average for all sending countries between 1950 and 2010 was 0.15 percent per year. Among the receiving countries, net migration rates for 1950 to 2010 were highest for Australia (0.73), Canada (0.56) and the US (0.29). The average for all receiving countries was 0.16 percent per year between 1950 and 2010. No study has considered both the capital cost and age structure effects of immigration in a steady-state framework, but SH Lee and Mason employed a dynamic model and concluded that the effects of immigration would likely be small if Korea were to greatly increase rates of immigration (*28*).

Very targeted immigration policies may yield economic benefits to residents. Countries maintain policies to attract highly-skilled workers, workers in occupations in short supply, and immigrants with capital with economic effects that our analysis does not address. Expanding immigration as a general solution to population aging does not appear to be an effective policy, however.

Table S1.

TFRs and fiscal support ratios (FSRs) for selected cases.

Country/ income group	TFR 2005-10	Replacement TFR	TFR that maximizes FSR	Stable FSR given TFR 2005-10	FSR given replacement TFR	Maximum stable FSR
All Countries	2.26	2.17	2.56	0.88	0.94	1.02
Lower income	3.98	2.37	1.08	1.04	1.20	1.39
India	2.66	2.34	1.80	1.00	1.01	1.01
Indonesia	2.50	2.16	0.88	1.04	1.09	1.23
Kenya	4.80	2.47	1.12	1.03	1.23	1.33
Mozambique	5.57	2.76	1.30	1.02	1.22	1.32
Philippines	3.27	2.16	1.13	1.06	1.18	1.25
Senegal	5.11	2.35	0.25	1.09	1.49	2.21
Upper middle income	2.05	2.16	2.96	0.87	0.87	0.96
Argentina	2.25	2.09	3.25	0.93	0.91	0.96
Brazil	1.90	2.13	5.45	0.71	0.74	0.93
China	1.63	2.25	2.64	0.81	0.85	0.85
Colombia	2.45	2.12	3.77	0.87	0.85	0.90
Costa Rica	1.92	2.08	3.85	0.72	0.74	0.83
Hungary	1.33	2.08	2.58	0.82	0.92	0.93
Mexico	2.37	2.10	2.83	0.91	0.89	0.92
Peru	2.60	2.15	3.45	0.82	0.77	0.85
South Africa	2.55	2.44	0.97	1.05	1.07	1.29
Thailand	1.49	2.12	0.79	1.06	0.95	1.14
High income	1.62	2.08	2.94	0.82	0.87	0.90
Austria	1.40	2.07	3.74	0.75	0.86	0.94
Chile	1.90	2.07	3.63	0.76	0.79	0.86
Finland	1.84	2.06	2.92	0.88	0.91	0.94
Germany	1.36	2.08	3.33	0.79	0.89	0.94
Japan	1.34	2.07	2.70	0.79	0.88	0.90
Slovenia	1.44	2.07	3.25	0.75	0.84	0.89
South Korea	1.23	2.09	2.07	0.82	0.85	0.85
Spain	1.41	2.08	3.29	0.71	0.80	0.84
Sweden	1.89	2.08	3.07	0.90	0.92	0.95
Taiwan	1.26	2.13	1.85	0.81	0.84	0.84
United Kingdom	1.88	2.07	3.00	0.91	0.92	0.95
United States	2.06	2.08	2.16	0.87	0.87	0.87
Uruguay	2.12	2.10	3.22	0.94	0.93	0.99

Sources: See sources for Table 1.

Notes: Results are calculated using the age-profiles of economic flows estimated for each country. All FSR values are normalized on the FSR based on the 2010 population age distribution. The "Stable" value is based on the fertility and survival schedule for 2005-10. The "Replacement fertility" value is based on the survival schedule and sex ratio at birth for 2005-10. The maximum support ratio is calculated using methods described in the theory section and the computer code included in the supplemental materials.

Table S2.

Support ratios under alternative conditions for 40 countries by income group.

Country/income group	Actual 2010	Stable for 2005-10 TFR	Replace-ment TFR	Maximum	Low capital cost case	High capital cost case
All Countries	**0.53**	**0.49**	**0.51**	**0.52**	**0.51**	**0.49**
Lower income	**0.52**	**0.52**	**0.58**	**0.61**	**0.59**	**0.55**
Cambodia	0.69	0.62	0.60	0.62	0.61	0.59
Ethiopia	0.44	0.48	0.57	0.60	0.58	0.54
Ghana	0.47	0.53	0.64	0.72	0.70	0.66
India	0.56	0.56	0.57	0.58	0.56	0.53
Indonesia	0.57	0.57	0.59	0.62	0.60	0.55
Kenya	0.41	0.42	0.48	0.48	0.47	0.46
Mozambique	0.49	0.49	0.58	0.61	0.60	0.57
Nigeria	0.41	0.42	0.56	0.64	0.62	0.56
Philippines	0.49	0.51	0.56	0.58	0.57	0.53
Senegal	0.60	0.63	0.73	0.75	0.72	0.62
Vietnam	0.58	0.49	0.50	0.51	0.50	0.48
Upper-middle income	**0.56**	**0.53**	**0.53**	**0.54**	**0.53**	**0.50**
Argentina	0.53	0.52	0.52	0.52	0.51	0.49
Brazil	0.51	0.45	0.46	0.46	0.45	0.43
China	0.54	0.46	0.46	0.47	0.46	0.44
Colombia	0.60	0.58	0.58	0.58	0.57	0.54
Costa Rica	0.56	0.51	0.51	0.51	0.50	0.48
Hungary	0.51	0.45	0.46	0.46	0.46	0.44
Jamaica	0.56	0.56	0.56	0.56	0.55	0.53
Mexico	0.56	0.55	0.56	0.56	0.55	0.52
Peru	0.54	0.53	0.53	0.53	0.52	0.50
South Africa	0.54	0.56	0.56	0.60	0.58	0.56
Thailand	0.60	0.50	0.51	0.51	0.50	0.48
Turkey	0.67	0.67	0.67	0.68	0.66	0.61
High income	**0.52**	**0.44**	**0.46**	**0.46**	**0.45**	**0.43**
Australia	0.56	0.48	0.49	0.49	0.48	0.47
Austria	0.53	0.42	0.46	0.46	0.45	0.44
Canada	0.56	0.48	0.48	0.48	0.47	0.46
Chile	0.57	0.51	0.51	0.51	0.50	0.48
Finland	0.49	0.44	0.45	0.45	0.44	0.43
France	0.46	0.41	0.41	0.42	0.41	0.40
Germany	0.49	0.39	0.43	0.44	0.43	0.42
Italy	0.53	0.44	0.46	0.46	0.46	0.44
Japan	0.47	0.38	0.42	0.42	0.41	0.40

Slovenia	0.45	0.36	0.38	0.38	0.37	0.36
South Korea	0.56	0.44	0.47	0.47	0.46	0.44
Spain	0.54	0.42	0.44	0.44	0.44	0.42
Sweden	0.47	0.43	0.43	0.43	0.43	0.41
Taiwan	0.56	0.42	0.45	0.45	0.44	0.43
United Kingdom	0.52	0.47	0.48	0.48	0.48	0.46
United States	0.54	0.48	0.48	0.48	0.47	0.45
Uruguay	0.51	0.51	0.51	0.51	0.50	0.48

Sources. See sources for Table 1.

Notes: Results are calculated using the age-profiles of economic flows estimated for each country. The "Actual" value is based on the 2010 population age distribution. The "Stable" value is based on the fertility and survival schedule for 2005-10. The "Replacement fertility" value is based on the survival schedule and sex ratio at birth for 2005-10. The maximum support ratio and the low and high cost ratios are calculated using methods described in the theory section and the computer code on the NTA website http://ntaccounts.org/web/nta/show/Science. The consumption maximizing values (low cost and high cost cases) use a depreciation rate of 5 percent per year, and exogenous labor-augmenting technological growth of 2 percent per year.

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